1. $a^m a^n = a^{m+n}$ (Why? $a^3 a^2 = (aaa)(aa) = aaaaa = a^{3+2}$)

2. $(a^m)^n = a^{mn}$ (Why? $(a^3)^2 = (aaa)(aaa) = aaaaaa = a^{3 \cdot 2}$)

3. $(ab)^m = a^m b^m$ (Why? $(ab)^3 = (ab)(ab)(ab) = ababab = aaabbb = (aaa)(bbb) = a^3 b^3$)

4. $a^0 = 1$ (Why? $a^0 = \frac{a^1}{a^1} a^0 = \frac{a^1 a^0}{a^1} = \frac{a^{1+0}}{a^1} = \frac{a^1}{a^1} = 1$)

5. $a^{-m} = \frac{1}{a^m}$ (Why? $a^{-m} = \frac{a^m}{a^m} a^{-m} = \frac{a^m a^{-m}}{a^m} = \frac{a^{m+(-m)}}{a^m} = \frac{a^0}{a^m} = \frac{1}{a^m}$)

6. $\frac{a^m}{a^n} = a^{m-n}$ (Why? $\frac{a^m}{a^n} = a^m \frac{1}{a^n} = a^m a^{-n} = a^{m-n}$)

7. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ (Why? $\left(\frac{a}{b}\right)^m = \left(a \cdot \frac{1}{b}\right)^m = (ab^{-1})^m = a^m (b^{-1})^m = a^m b^{-1 \cdot m} = a^m b^{-m} = \frac{a^m}{b^m}$)

8. $\sqrt{a} = a^{\frac{1}{2}}$ (Why? $(\sqrt{a})^2 = a = a^1 = a^{\frac{1}{2} \cdot 2} = (a^{\frac{1}{2}})^2$)

9. $\sqrt[n]{a} = a^{\frac{1}{n}}$ (Why? similar: $(\sqrt[n]{a})^n = a = a^1 = a^{\frac{1}{n} \cdot n} = (a^{\frac{1}{n}})^n$)

10. $\sqrt[n]{a^m} = (\sqrt[n]{a})^m = a^{\frac{m}{n}}$ (Why? $\sqrt[n]{a^m} = (a^m)^{\frac{1}{n}} = a^{m \cdot \frac{1}{n}} = a^{\frac{m}{n}} = a^{\frac{1}{n} \cdot m} = (a^{\frac{1}{n}})^m = (\sqrt[n]{a})^m$)

11. $\sqrt[n]{ab} = \sqrt[n]{a} \sqrt[n]{b}$ (Why? $\sqrt[n]{ab} = (ab)^{\frac{1}{n}} = a^{\frac{1}{n}} b^{\frac{1}{n}} = \sqrt[n]{a} \sqrt[n]{b}$)

12. $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ (Why? $\sqrt[n]{\frac{a}{b}} = \left(\frac{a}{b}\right)^{\frac{1}{n}} = \frac{a^{\frac{1}{n}}}{b^{\frac{1}{n}}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$)

Not exponent properties, but useful:

1. $\frac{ab}{ac} = \frac{b}{c}$ (simplifying fractions)

2. $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$ (adding fractions with like denominators)

3. $\frac{a}{c} + \frac{b}{d} = \frac{ad+bc}{cd}$ (Why? $\frac{a}{c} + \frac{b}{d} = \frac{a}{c} \cdot \frac{d}{d} + \frac{b}{d} \cdot \frac{c}{c} = \frac{ad}{cd} + \frac{bc}{cd} = \frac{ad+bc}{cd}$)

4. $\frac{a}{c} \cdot \frac{b}{d} = \frac{ab}{cd}$ (multiplying fractions)

5. $\frac{1}{\frac{a}{b}} = \frac{b}{a}$ (reciprocals) (Why? $\frac{1}{\frac{a}{b}} = \frac{\frac{b}{a}}{\frac{b}{a}} \cdot \frac{1}{\frac{a}{b}} = \frac{\frac{b}{a}}{\frac{b}{a} \cdot \frac{a}{b}} = \frac{\frac{b}{a}}{\frac{ba}{ab}} = \frac{\frac{b}{a}}{1} = \frac{b}{a}$)

The following inequalities represent things that students sometimes erroneously think should be equal, but are usually not equal. I am intentionally misusing the $\neq$ sign here to mean "is *usually* not equal to," when it actually means "is not equal to."

1. $\frac{a+b}{a+c} \neq \frac{b}{c}$ (e.g. $\frac{1+2}{1+3} = \frac{3}{4} \neq \frac{2}{3}$)

2. $(a+b)^m \neq a^m + b^m$ (e.g. $(1+3)^2 = 4^2 = 16 \neq 10 = 1^2 + 3^2$)

3. $a^{-m} \neq -a^m$ (e.g. $2^{-3} = \frac{1}{2^3} = \frac{1}{8} \neq -8 = -(2^3) = -2^3$)